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Dynamics of an M-Level Atom Interacting with Cavity Fields:  
Properties of Photon Statistics

by

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Dynamics of an M-level atom interacting with cavity fields:  
Properties of photon statistics

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Abstract

For a system of an M-level atom interacting with cavity fields, the statistical properties of the field are investigated numerically. The variation of photon antibunching and probability distribution with the atomic level number and initial field intensity are discussed for both resonance and off-resonance cases.

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## I. Introduction

There are two aspects of the problem of an atom interacting with radiation fields in a cavity, the dynamical behavior of the atom during the interaction process and the evolution of the coherence and statistical properties of the radiation field. Although a large amount of work on atomic properties has been carried out in the Jaynes-Cummings (JC) model,<sup>1</sup> relatively less attention has been paid to the field properties.

Early studies of the first-order correlation function in the JC model revealed that the disparity between  $\langle a^\dagger(t)a(t) \rangle$  and  $\langle a^\dagger(t) \rangle \langle a(t) \rangle$  increases with time if the field is initially in the coherent state.<sup>2</sup> Diagonal elements of the field density matrix were explicitly calculated later.<sup>3</sup> Only recently were effects on the nonclassical behavior of the atom and the field due to the cavity loss discussed.<sup>4</sup> The upper and lower bounds of the Q-value in the JC model were given analytically,<sup>5</sup> and an analysis of these bounds show that the larger the amplitude of the initial coherent state, the less sub-Poisson (or super-Poisson) the photon statistics can become.

A direct generalization of the JC model is the system of a three-level atom interacting with one-mode or two-mode cavity fields. The off-resonance behavior of the cavity fields has been investigated by two of the present authors and others.<sup>6,7</sup> It was discovered in Ref. 6 that the initially coherent field does not lose its coherence right after the interaction with the atom takes place when the interaction is off resonance. Instead, it may recover coherence almost periodically. As a matter of fact, the field can remain coherent for a long time if the detunings are sufficiently far away from the resonance. The photon probability distribution and fluctuation are discussed in detail in Ref. 7, where it is found that for a one-mode  $\Lambda$ -type atom, sub-Poisson and super-Poisson distributions appear alternatively as time

develops, and for a two-mode  $\Lambda$ -type the distribution is always super-Poisson. Furthermore, photons involved in the former case exhibit antibunching phenomena and in the latter case exhibit bunching phenomena.

The photon statistics has also been studied in another case in which an atom with one common upper and  $N-1$  lower levels interacts with  $N-1$  modes of a cavity field.<sup>8</sup> A numerical calculation was performed for resonance interaction with initially one mode (called the pumping mode) of coherent light present. Once the interaction takes place, all the other modes (called signal modes) start to build up. Photons in these signal modes are found to show sub-Poisson distribution and hence exhibit antibunching all the time.

As the observation of the one-atom maser has become practical,<sup>9</sup> and since the atom is excited to high levels with principal quantum number in the range of 30 ~ 40, it is inevitable that multiphoton processes are involved. In fact, a micromaser based on degenerate two-photon transitions is already realized.<sup>10</sup> Theoretically, however, there does not seem to have been sufficient attention given to multiphoton processes in the literature thus far. In two recent papers,<sup>11,12</sup> the present authors considered a single  $M$ -level atom interacting with cavity fields with arbitrary detunings. Time evolution of atomic level occupation probabilities, mean photon number and field squeezing have been investigated in detail. Their dependence on the atomic level number as well as on the initial field intensity has been discussed. Here in this paper, we discuss the photon probability distribution and fluctuation and their variations with various parameters.

An outline of the theory is given in Sec. II. Results of the numerical calculation are presented and discussed in Sec. III, and concluding remarks are given in Sec. IV.

## II. Theory

The general formalism of the theory can be found in Ref. 11 (hereafter referred to as I), and we only outline the essentials here. We consider a cascade atom with  $M$  energy levels as shown in Fig. 1. The total Hamiltonian is, in the rotating wave approximation,

$$H = \hbar\Omega a^\dagger a + \sum_{i=1}^M \hbar\omega_i A_i^\dagger A_i + \sum_{i=1}^{M-1} \lambda_i A_{i+1}^\dagger A_i a + \text{h.c.} \quad (1)$$

where the operator  $a^\dagger$  creates a photon,  $A_i^\dagger$  creates an atom in the  $i$ -th level, and  $\lambda_i$  are the atom-field coupling constants. Our discussion will be limited, for simplicity, to processes involving only a one-photon transition between adjacent atomic levels.

We define the total excitation number operator

$$\hat{N} = a^\dagger a + \sum_{i=1}^M i A_i^\dagger A_i, \quad (2)$$

which is the sum of the photon number and atomic excitation number operators.

It can readily be verified that  $[\hat{H}, \hat{N}] = 0$ , i.e.,  $\hat{N}$  is a constant of motion.

If we are only interested in the particular case in which the atom is initially in its highest level, then we need only consider that part of the Hilbert space involving state vectors corresponding to  $N \geq M$ , where  $N$  is the eigenvalue of the operator  $\hat{N}$ . This is a consequence of the fact that  $N$  is a conserved quantity.

Consider the state

$$|i, n\rangle = |i\rangle |n\rangle \quad (3)$$

in which there are  $n$  photons in the field and the atom is in the  $i$ -th state. The subspace corresponding to  $N = M + n$  is spanned by the state vectors  $|M, n\rangle$ ,  $|M-1, n+1\rangle$ , ...,  $|i, n+M-i\rangle$ , ...,  $|1, n+M-1\rangle$ , where  $n$  is the photon number in the field and can take any positive integral value. Since state vectors in different subspaces are necessarily orthogonal, the total Hamiltonian can be diagonalized in every subspace.

An arbitrary state in the subspace corresponding to  $N$  can be expressed as

$$|\phi_n\rangle = \sum_{i=1}^M C_{i, n+M-i} |i, n+M-i\rangle, \quad (4)$$

where the expansion coefficients  $C_{i, n+M-i}$  satisfy the stationary-state Schrödinger equation

$$\sum_{i'=1}^M (H_{ii'} \delta_{ii'} + H_{ii'} - E \delta_{ii'}) C_{i', n+M-i'} = 0. \quad (5)$$

The matrix elements in (5) are given by

$$H_{ii} = \langle i, n+M-i | H | i, n+M-i \rangle = (n+M-1)\hbar\omega + \hbar\omega_1 - \sum_{j=1}^{i-1} \Delta_j \quad (5a)$$

$$H_{ii'} = \langle i, n+M-i | H | i', n+M-i' \rangle$$

$$= \lambda_{i, \sqrt{n+m-i'}} \delta_{i, i'+1} + \lambda_{i, -1} \sqrt{n+M-i'+1} \delta_{i, i'-1} \quad , \quad i \neq i' \quad (5b)$$

Here we have defined the detuning parameters

$$\Delta_i = \hbar\Omega - \hbar(\omega_{i+1} - \omega_i) \quad , \quad i = 1, 2, \dots, M-1 \quad . \quad (6)$$

The energy eigenvector  $|\phi_{n\sigma}\rangle$  and the corresponding eigenvalue  $E_{n\sigma}$  are determined by

$$H|\phi_{n\sigma}\rangle = E_{n\sigma}|\phi_{n\sigma}\rangle \quad (7a)$$

$$|\phi_{n\sigma}\rangle = \sum_{i=1}^M c_{i, n+M-1}^{\sigma} |i, n+M-i\rangle \quad . \quad (7b)$$

The subscript  $\sigma$  in (7a) labels the eigenstates in the subspace in question.

If the atom is assumed to be initially in the  $M$ -th level, then for all time  $t > 0$  the system can only be found in subspaces with  $N \geq M$ . There are  $M$  eigenstates of the type (7b) in every subspace. Hence the orthonormality condition is

$$\sum_{i=1}^M c_{i, n+M-i}^{\sigma'} c_{i, n+M-i}^{\sigma} = \delta_{\sigma'\sigma} \quad , \quad (8)$$

and the completeness relation is

$$\sum_{n=0}^{\infty} \sum_{\sigma=1}^M |\phi_{n\sigma}\rangle \langle \phi_{n\sigma}| = 1 \quad . \quad (9)$$



We can now solve the equation of motion for the density matrix. Its matrix element between energy eigenstates satisfies the equation

$$\frac{\partial}{\partial t} \rho_{n\sigma, n'\sigma'}(t) = -\frac{i}{\hbar} (E_{n\sigma} - E_{n'\sigma'}) \rho_{n\sigma, n'\sigma'} \quad (10)$$

whose solution can be put in the form

$$\rho_{n\sigma, n'\sigma'}(t) = \rho_{n\sigma, n'\sigma'}(0) e^{-\frac{i}{\hbar} (E_{n\sigma} - E_{n'\sigma'}) t} \quad (11)$$

The initial matrix element in (11) is given by

$$\rho_{n\sigma, n'\sigma'}(0) = \rho_{nn'} (C^{-1})_{\sigma, n+M-\sigma}^M (C^{-1})_{\sigma', n+M-\sigma'}^M \quad (12)$$

where  $C^{-1}$  is the inverse matrix of the transformation defined in (4) and  $\rho_{nn'}$  is the initial density matrix element of the field.

The normalized intensity correlation function of the field is defined as

$$g^{(2)}(t) = \langle a^{\dagger 2} a^2 \rangle / \langle a^{\dagger} a \rangle^2 \quad (13)$$

where

$$\langle n \rangle = \langle a^{\dagger} a \rangle = \text{Tr}(\rho a^{\dagger} a)$$

$$= \sum_{n=0}^{\infty} \sum_{i=1}^M (n+M-i) \sum_{\sigma=1}^M \sum_{\sigma'=1}^M C_{i, n+M-i}^{\sigma} C_{i, n+M-i}^{\sigma'}$$

$$\times \rho_{n\sigma, n\sigma'}(0) \cos\left(\frac{(E_{n\sigma} - E_{n\sigma'})t}{\hbar}\right) \quad (14)$$

$$\begin{aligned} \langle a^{\dagger 2} a^2 \rangle &= \sum_{n=0}^{\infty} \sum_{i=1}^M (n+M-i)(n+M-i-1) \\ &\times \sum_{\sigma=1}^M \sum_{\sigma'=1}^M C_{i, n+M-i}^{\sigma} C_{i, n+M-i}^{\sigma'} \rho_{n\sigma, n\sigma'}(0) \cos\left(\frac{(E_{n\sigma} - E_{n\sigma'})t}{\hbar}\right) \quad (15) \end{aligned}$$

The probability of finding  $m$  photons in the field at time  $t$  is given by

$$p_m(t) = \sum_{n=m-M+1}^m \langle i, n+M-i | \rho | i, n+M-i \rangle, \quad (16)$$

where  $i = n+M-m$  and  $n \geq 0$ . Taking the inverse transformation of (7b) and plugging it into (16), we find

$$\begin{aligned} p_m(t) &= \sum_{n=m-M+1}^m \sum_{j=1}^M \sum_{j'=1}^M (C^{-1})_{j', n+M-j}^{n+M-m} (C^{-1})_{j, n+M-j}^{n+M-m} \\ &\times \rho_{nj, nj'}(0) \cos\left(\frac{(E_{nj} - E_{nj'})t}{\hbar}\right), \quad (17) \end{aligned}$$

where we have made use of (11). It is then not difficult to prove that

$$\sum_{m=0}^{\infty} p_m(t) = \sum_{n=0}^{\infty} \sum_{j=1}^M \rho_{nj, nj}(0) = 1. \quad (18)$$

The statistical properties of the field are completely described by Eqs. (13) and (17), which we shall discuss in the following section.

### III. Results and discussion

Throughout this paper, we take  $M = 1$  in our numerical calculation. The above formulas are derived on the assumption the atom is initially in its highest level  $M$ .<sup>11</sup> We now assume further that the field is initially in the coherent state with mean photon number  $\bar{n}$ . Thus the diagonal matrix element of the initial density matrix of the field is

$$\rho_{nn} = \frac{\bar{n}^n}{n!} e^{-\bar{n}} \quad (19)$$

As is well known, when the field is strictly in the coherent state, the equal-time second-order correlation function is  $g^{(2)}(t) = 1$ . When  $g^{(2)}(t) < 1$ , the field exhibits antibunching, and when  $g^{(2)}(t) > 1$ , the field exhibits bunching. The time evolution of the correlation function calculated from (13) is presented in Figs. 2 and 3 for various  $M$  and  $\bar{n}$ . Resonance conditions  $\Delta_i = 0$  have been assumed here for simplicity. It is observed that the correlation function oscillates irregularly around its initial value of 1. Thus the field exhibits alternatively bunching and antibunching as time develops. For a fixed  $M$ , this oscillation amplitude decreases with the increasing  $\bar{n}$ . In other words, the stronger the initial field is, the weaker the effect of bunching or antibunching becomes. Therefore, strong photon bunching or antibunching results if the initial field strength weakens. The same conclusions have been obtained for the  $M = 2$  case.<sup>5</sup>

Physically, it is not difficult to understand this phenomenon. Since we have assumed that the field is in the coherent state initially, at any later time or  $t > 0$ , the total field is the sum of the initial field plus the radiation field of the stimulated atom. The latter acts like a modulator that causes the initial field to deviate from coherence. The smaller the  $\bar{n}$ , the easier the initial field is modulated. At any time  $t > 0$ , the total field is dominated by the stimulated radiation field which is an oscillating function of time. As a consequence, the total field oscillates between bunching and antibunching. If we compare the curves for the same  $\bar{n}$  but different  $M$ , we find that the effect of antibunching decreases when the number of atomic levels increases. Similar results have also been found for a common upper  $N$ -level atom interacting with  $N-1$  modes of cavity fields.<sup>8</sup>

The photon number fluctuation is directly related to the  $Q$ -parameter defined as<sup>13</sup>

$$Q = \frac{\langle n^2 \rangle - \langle n \rangle^2}{\langle n \rangle} - 1 \quad (20)$$

When  $Q > 0$ , the fluctuation is large and the photon probability distribution is super-Poisson. When  $Q < 0$ , the fluctuation is small and the probability distribution is sub-Poisson. The lower bound of  $Q$  is  $-1$ , corresponding to zero fluctuation, where the probability has its sharpest peak. When  $Q = 0$ , the photon probability has a Poisson distribution, indicating a coherent state of the field. The time evolution of the  $Q$ -parameter is presented in Figs. 6 and 7 for different  $M$  and  $\bar{n}$ . From these figures we can see that, similar to  $g(2)$ , for fixed  $M$  the amplitude of the oscillation of  $Q$  becomes smaller when  $\bar{n}$  increases, and that the photon probability approaches the Poisson distribution

when  $\bar{n}$  increases. For fixed  $\bar{n}$ , increasing  $M$  will cause the photon probability to deviate from the Poisson distribution and tend to become super-Poisson.

Since the  $Q$ -parameter is related to the normalized Glauber correlation function  $g^{(2)}(0)$  by

$$Q = \langle n \rangle [g^{(2)}(0) - 1] \quad , \quad (21)$$

it is clear that photons exhibit the antibunching phenomenon whenever the probability distribution is sub-Poisson, and exhibits bunching phenomenon when the probability has a super-Poisson distribution. It should be noted that a minimum  $g^{(2)}$  does not imply a minimum  $Q$  because of the factor  $\langle n \rangle$  in (21), except that  $\langle n \rangle$  is just in the time region with collapsed oscillation. This can be seen by comparing Figs. 2, 4, and 6 or Figs. 3, 5 and 7 with one another. We have also found that for certain  $\bar{n}$  and  $M$  the field can be kept in a coherent state for a long time with  $g^{(2)} \approx 1$  and  $Q \approx 0$  (see Figs. 2(c) and 6(c)). This means that the initially coherent field can recover its coherence after the interaction with the atom takes place. This is in agreement with the conclusions of Ref. 6.

Next we look at the case in which the detunings are nonzero. We assume that  $|\Delta_i| \neq 0$  ( $i = 1, 2, \dots, M-1$ ) but  $\sum_{i=1}^M \Delta_i = 0$ . That is, multiphoton transitions involving less than  $M-1$  photons are all detuned, and only the  $(M-1)$ -fold degenerate photon transition satisfies the resonance condition. The correlation function is calculated for  $M = 3$  and  $9$ ,  $\bar{n} = 3$  and  $|\Delta_i| = 0.5$  and  $1.5$ . The results are plotted in Figs. 8 and 9. We see from these figures that for a given  $M$ , a larger detuning results in weaker antibunching but stronger bunching. This feature becomes even stronger as  $M$  becomes larger. On the other hand, when the detunings  $|\Delta_i|$  are fixed, then larger  $M$

corresponds to weaker antibunching. This means that the more photons that are involved in the degenerate multiphoton transition, the weaker the antibunching becomes in the process.

In addition, we have also examined the sub-Poisson distributions of the photons with the same  $M$  but different  $\bar{n}$ , where we find sharper peaks in the distribution when the initial field strength is weaker or when  $\bar{n}$  is smaller.

oted, however, that  $\bar{n}$  should be kept above a certain value, say,  $\geq 3$ ,

otherwise the distribution approaches the shape of a thermal distribution. We have also computed the photon distribution for nonresonant cases (numerical results are not presented here), where we have found that the detunings tend to weaken the sub-Poisson and super-Poisson effects.

#### IV. Conclusions

We have shown that multiphoton processes lead to a minimum fluctuation in the field intensity. Although such a markedly reduced fluctuation occurs only at some particular time, it can be achieved by adjusting the velocity of the atom injected into the optical cavity, because the JC model is realized in practice by the micromaser.<sup>9</sup> In other words, the most stable intensity of the output field can be obtained by adjusting the duration that the atom stays within the cavity. This is also true for the squeezing of the field.<sup>12</sup>

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## Figure captions

1. Schematic diagram of the atomic levels.
2. Time evolution of the correlation function  $g^{(2)}$  under resonance conditions  $\Delta_i = 0$  for  $M = 3$ : (a)  $\bar{n} = 3$ , (b)  $\bar{n} = 5$ , (c)  $\bar{n} = 10$ .
3. Same as Fig. 2 except  $M = 9$ .
4. Time evolution of the mean photon number  $\langle n \rangle$  for  $\Delta_i = 0$  and  $M = 3$ : (a)  $\bar{n} = 3$ , (b)  $\bar{n} = 5$ , (c)  $\bar{n} = 10$ .
5. Same as Fig. 4, except  $M = 9$ .
6. Time evolution of Mandel's Q-parameter for  $\Delta_i = 0$  and  $M = 3$ : (a)  $\bar{n} = 3$ , (b)  $\bar{n} = 5$ , (c)  $\bar{n} = 10$ .
7. Same as Fig. 6, except  $M = 9$ .
8. Time evolution of  $g^{(2)}$  for  $M = 3$  and  $\bar{n} = 3$ . The resonance conditions are not satisfied by the individual transitions, but  $\sum_i \Delta_i = 0$ , where  $\Delta_1, \dots, \Delta_{\frac{M-1}{2}} > 0$ ,  $\Delta_{\frac{M+1}{2}}, \dots, \Delta_{M-1} < 0$  ( $i = 1, \dots, M-1$ ): (a)  $|\Delta_i| = 0.5$ , (b)  $|\Delta_i| = 1.5$ .
9. Same as Fig. 8 except  $M = 9$ .

Fig. 1

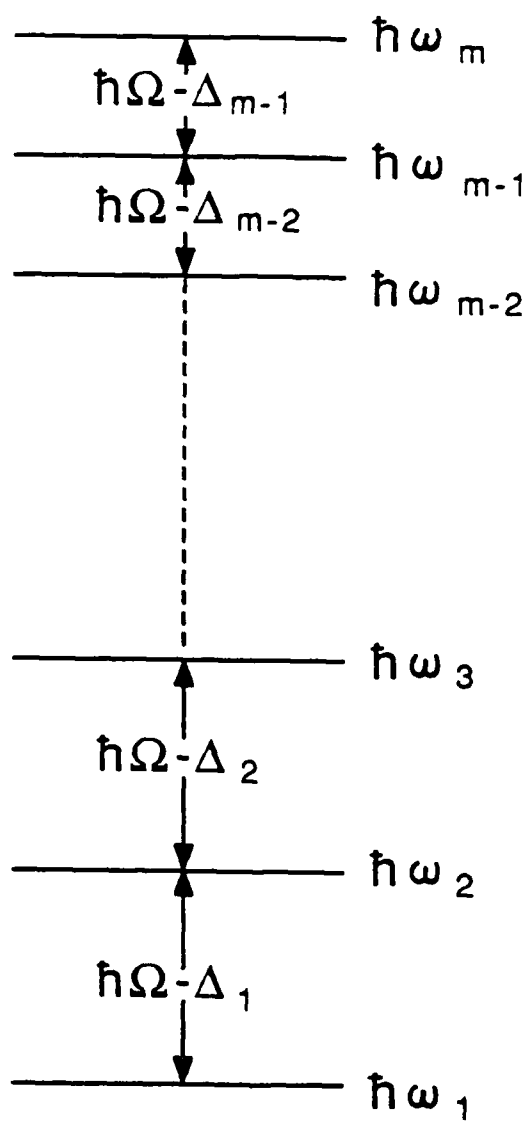


Fig 2

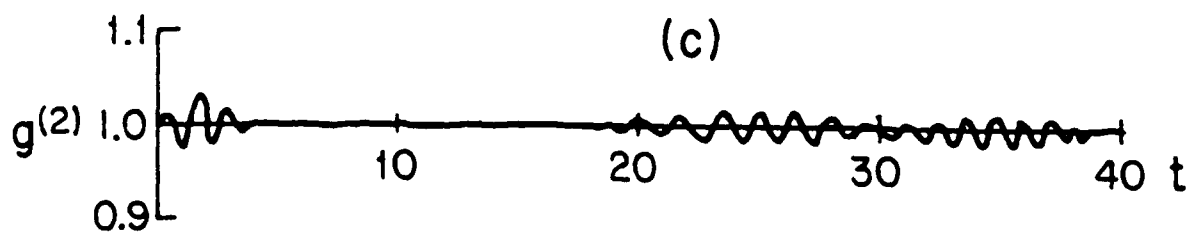
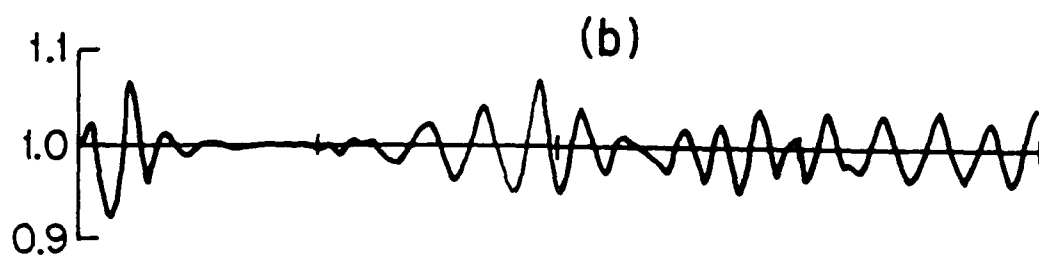
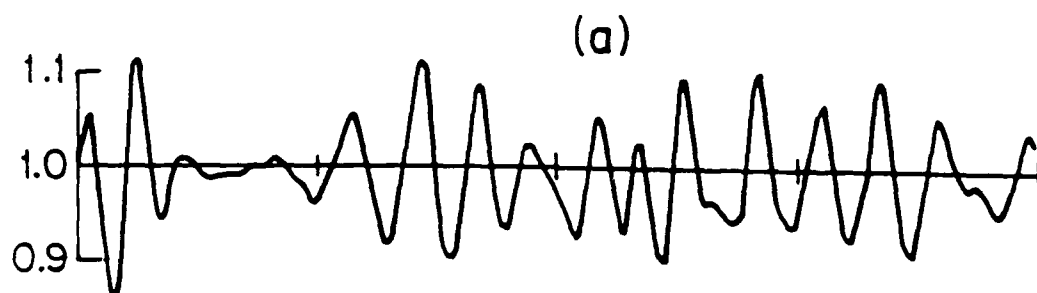


Fig. 3

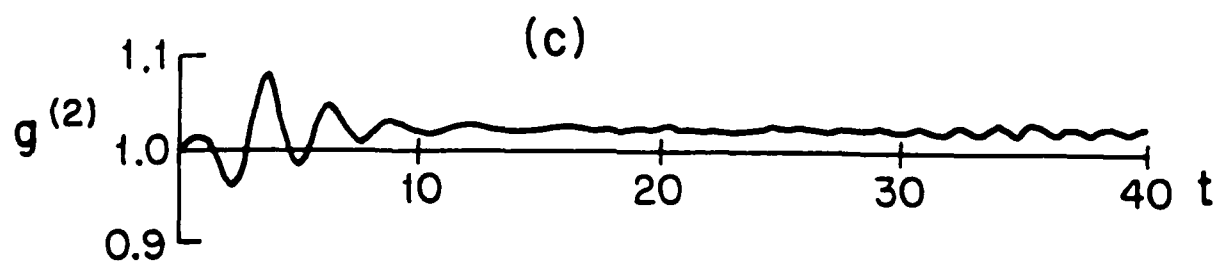
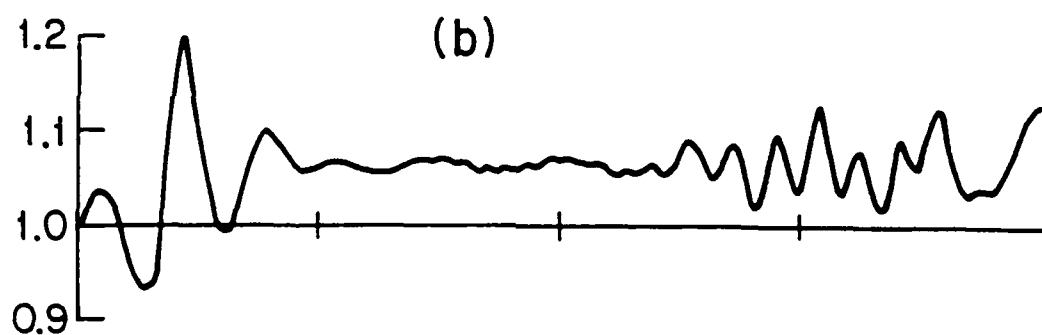
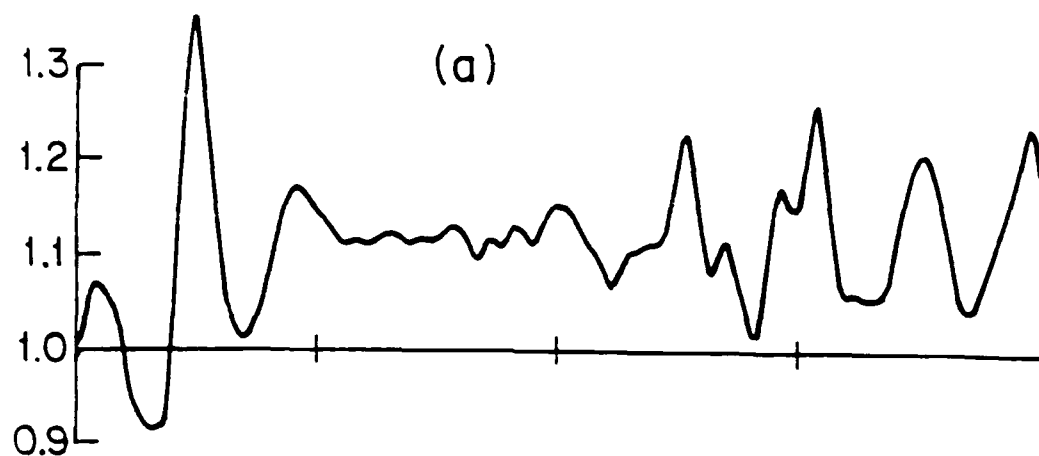


Fig. 4

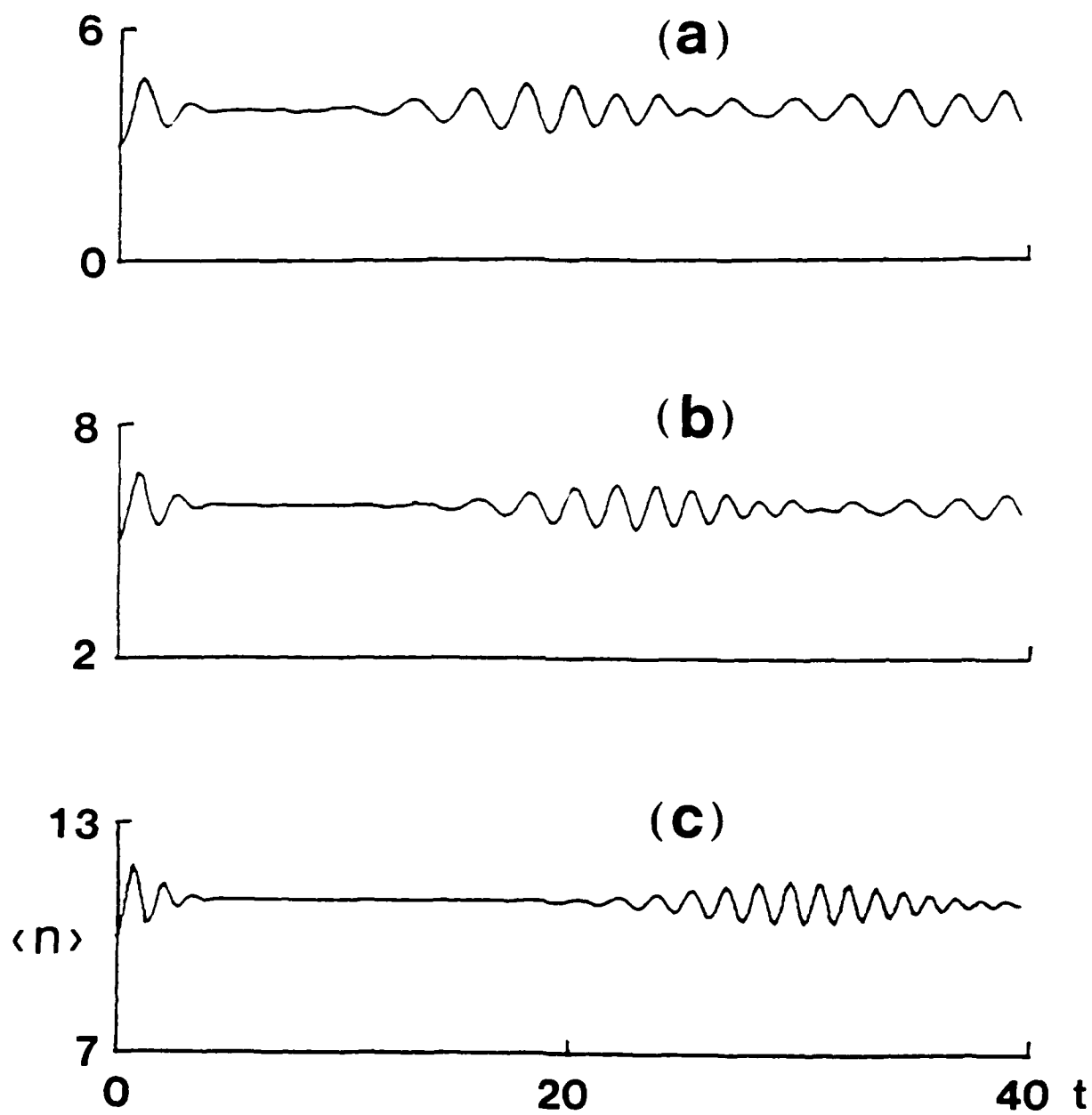


Fig. 5

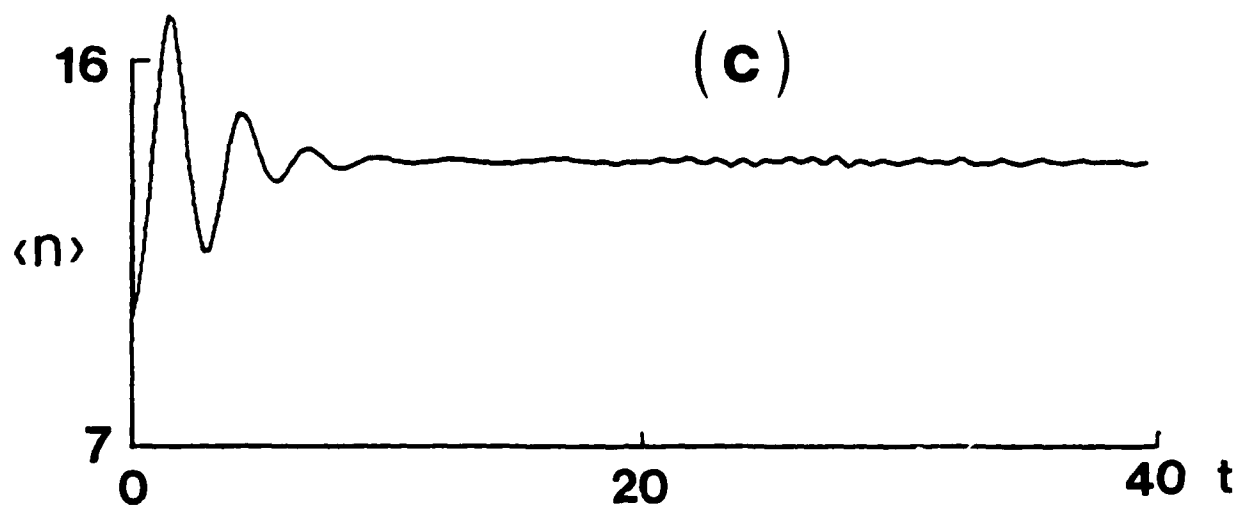
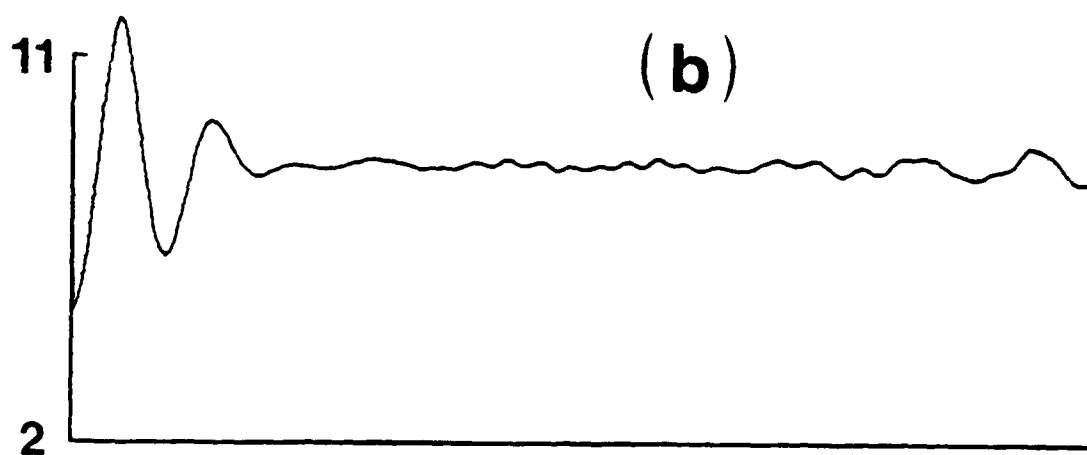
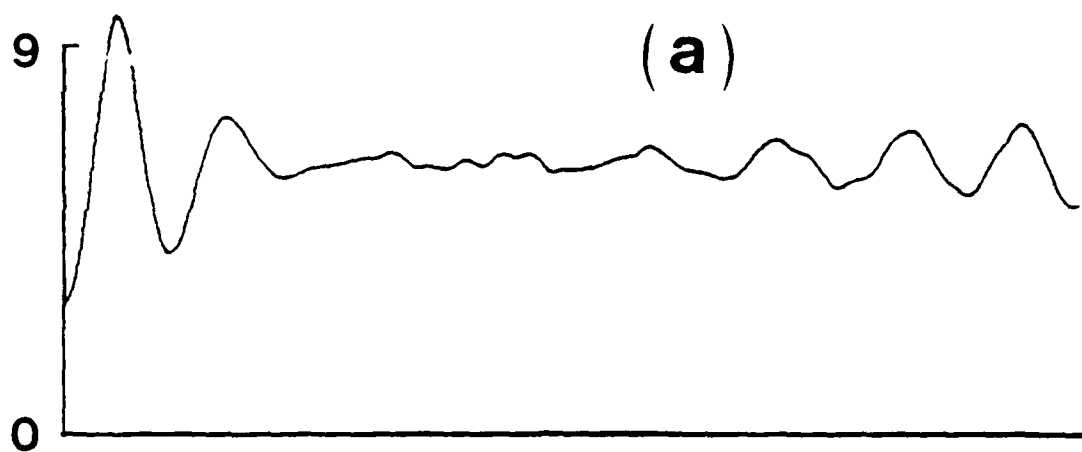


Fig. 6

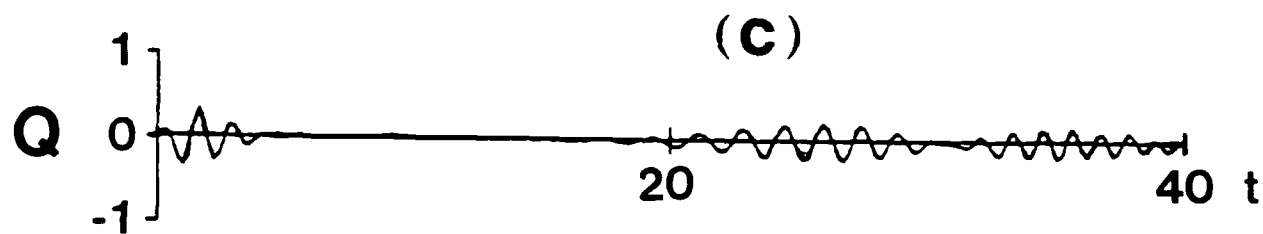
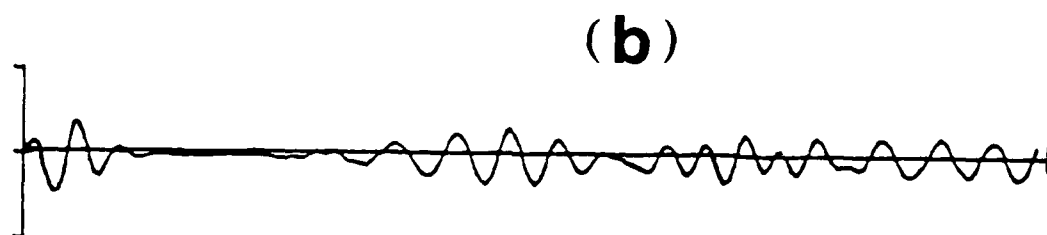
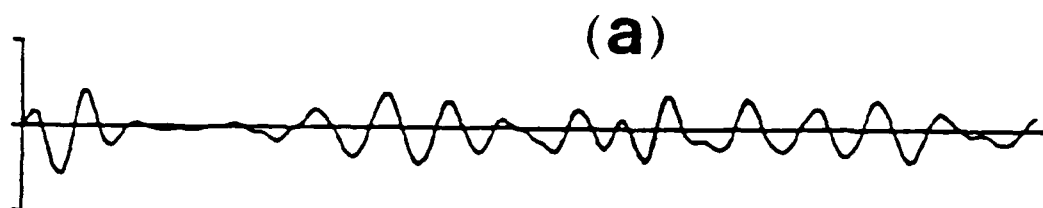


Fig 7

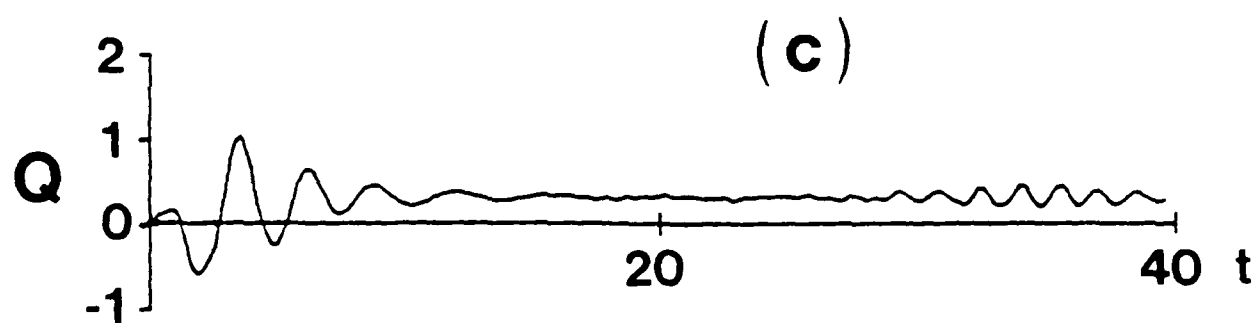
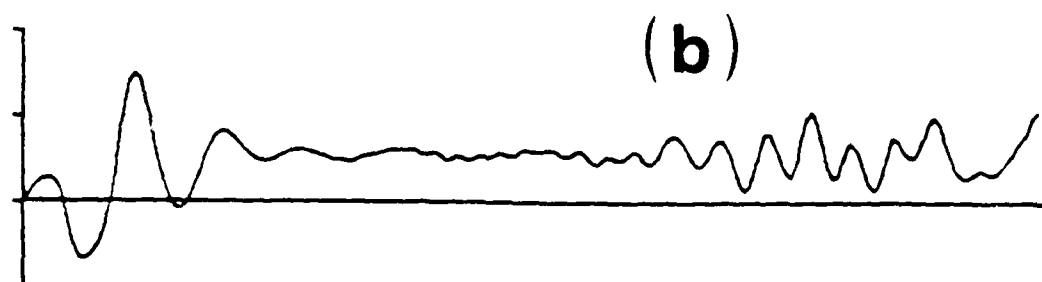
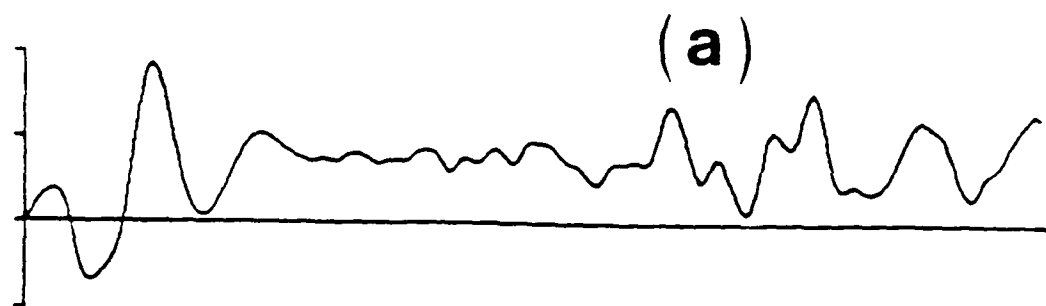




Fig. 8

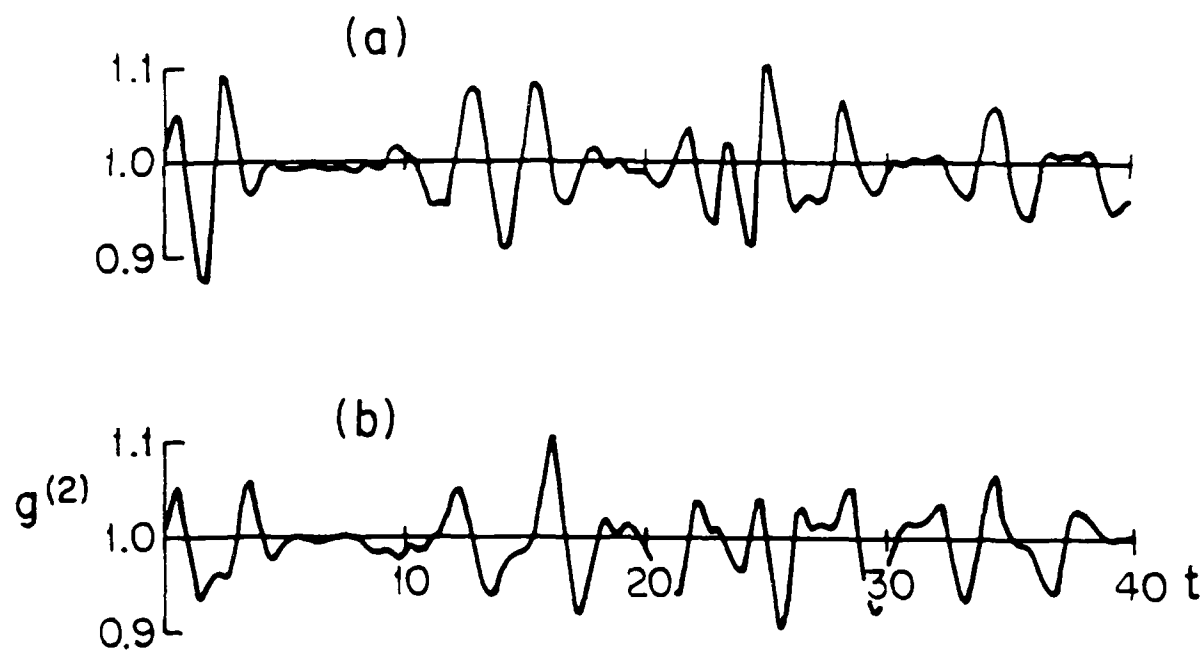
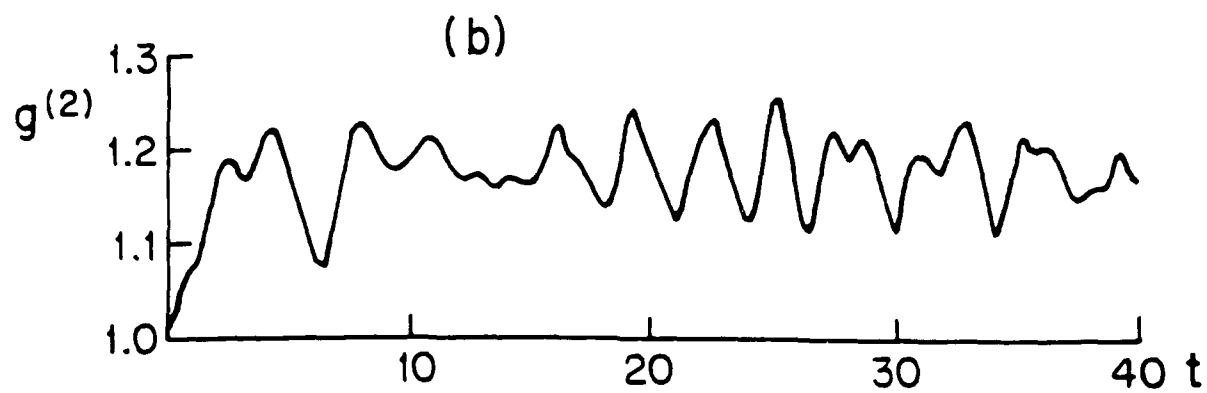
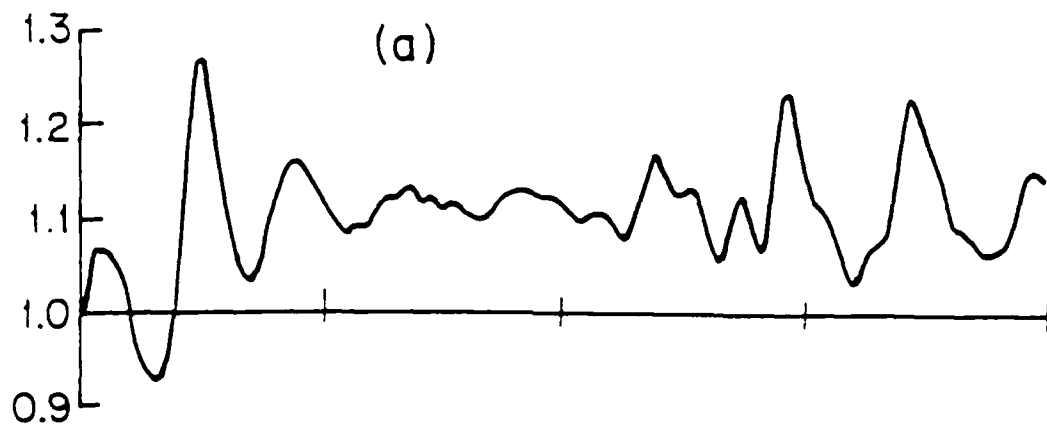


Fig. 9



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